Digital Image Processing

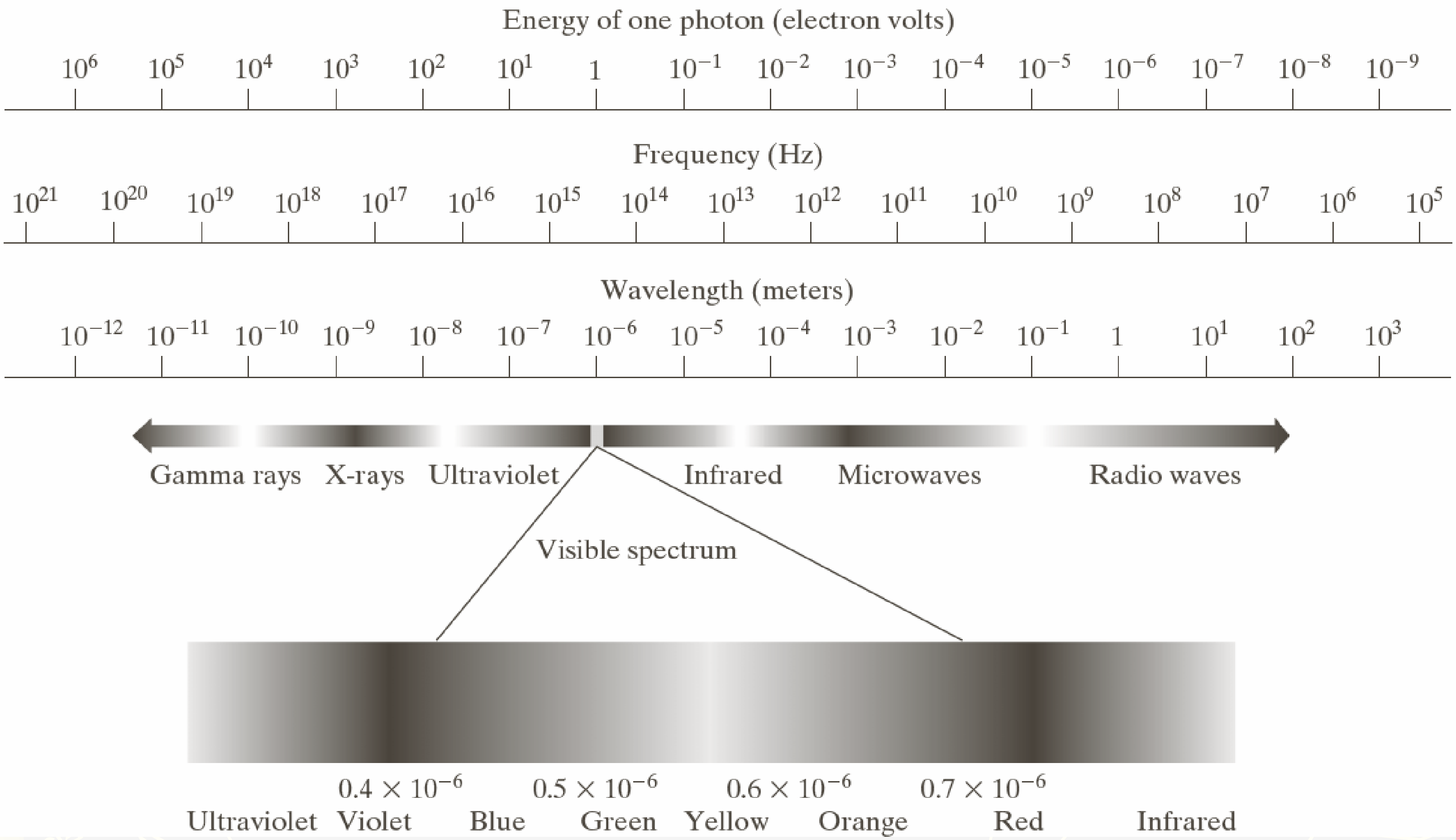
# Lecture 2
# Fundamentals

Autumn 2010

# Agenda

- Light and EM spectrum
- Image acquisition
- Image formation model
- Sampling and quantization
- Spatial and intensity resolution
- Zooming and shrinking
- Basic relationship b/w pixels

# Light and EM Spectrum




$$c = \lambda \nu \qquad E = h\nu, \quad h: \text{ Planck's constant.}$$

# Light and EM Spectrum

- The colors that humans perceive in an object are determined by the nature of the light reflected from the object.

  e.g. green objects reflect light with wavelengths primarily in the 500 to 570 nm range while absorbing most of the energy at other wavelength

# Light and EM Spectrum

- Monochromatic light: void of color

  **Intensity** is the only attribute, from black to white

  Monochromatic images are referred to as **gray-scale** images

- Chromatic light bands: 0.43 to 0.79 um

  The quality of a chromatic light source:

  **Radiance**: total amount of energy

  **Luminance (lm)**: the amount of energy an observer perceives from a light source

  **Brightness**: a subjective descriptor of light perception that is impossible to measure. It embodies the achromatic notion of intensity and one of the key factors in describing color sensation.

# Image Acquisition

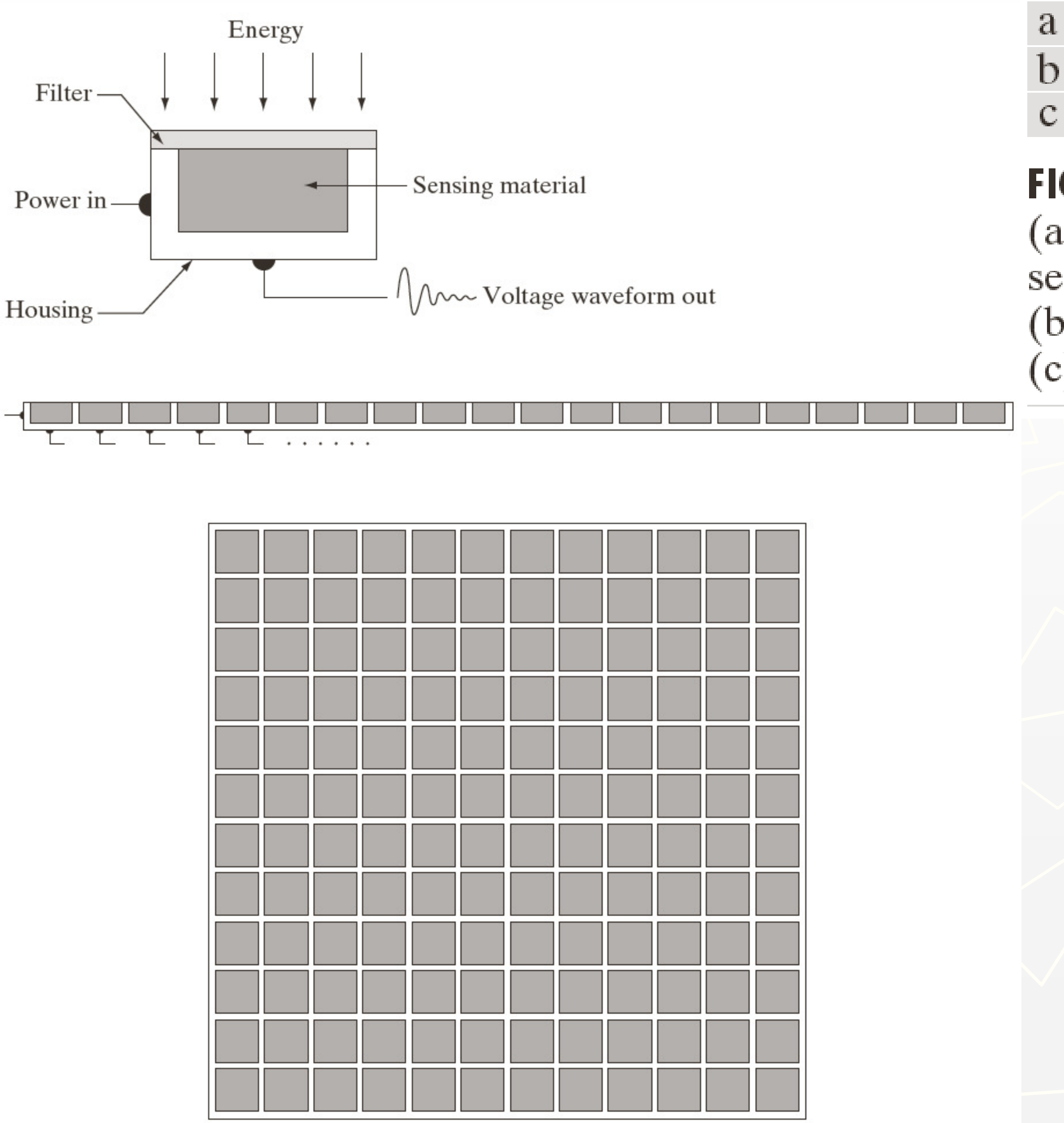
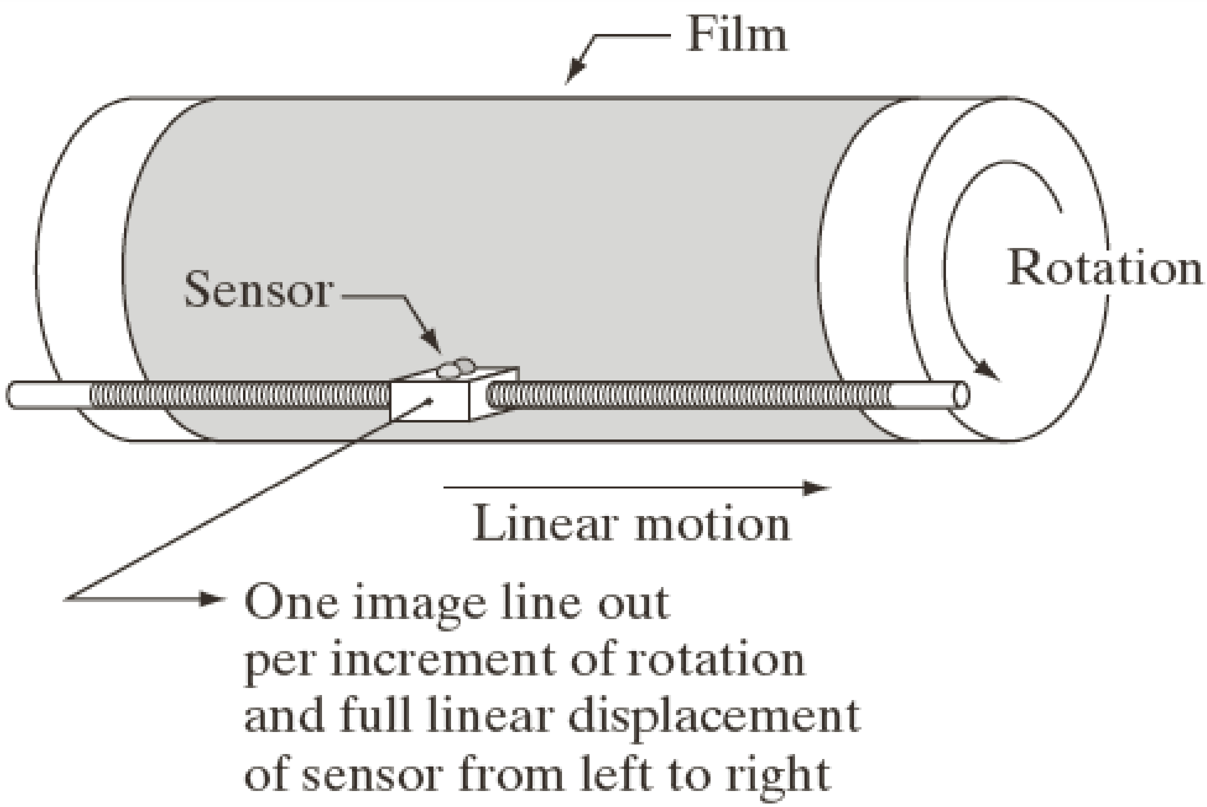



**FIGURE 2.12**
(a) Single imaging sensor.
(b) Line sensor.
(c) Array sensor.

Transform illumination energy into digital images

# Image Acquisition Using a Single Sensor

**FIGURE 2.13**
Combining a single sensor with motion to generate a 2-D image.



# Image Acquisition Using Sensor Strips

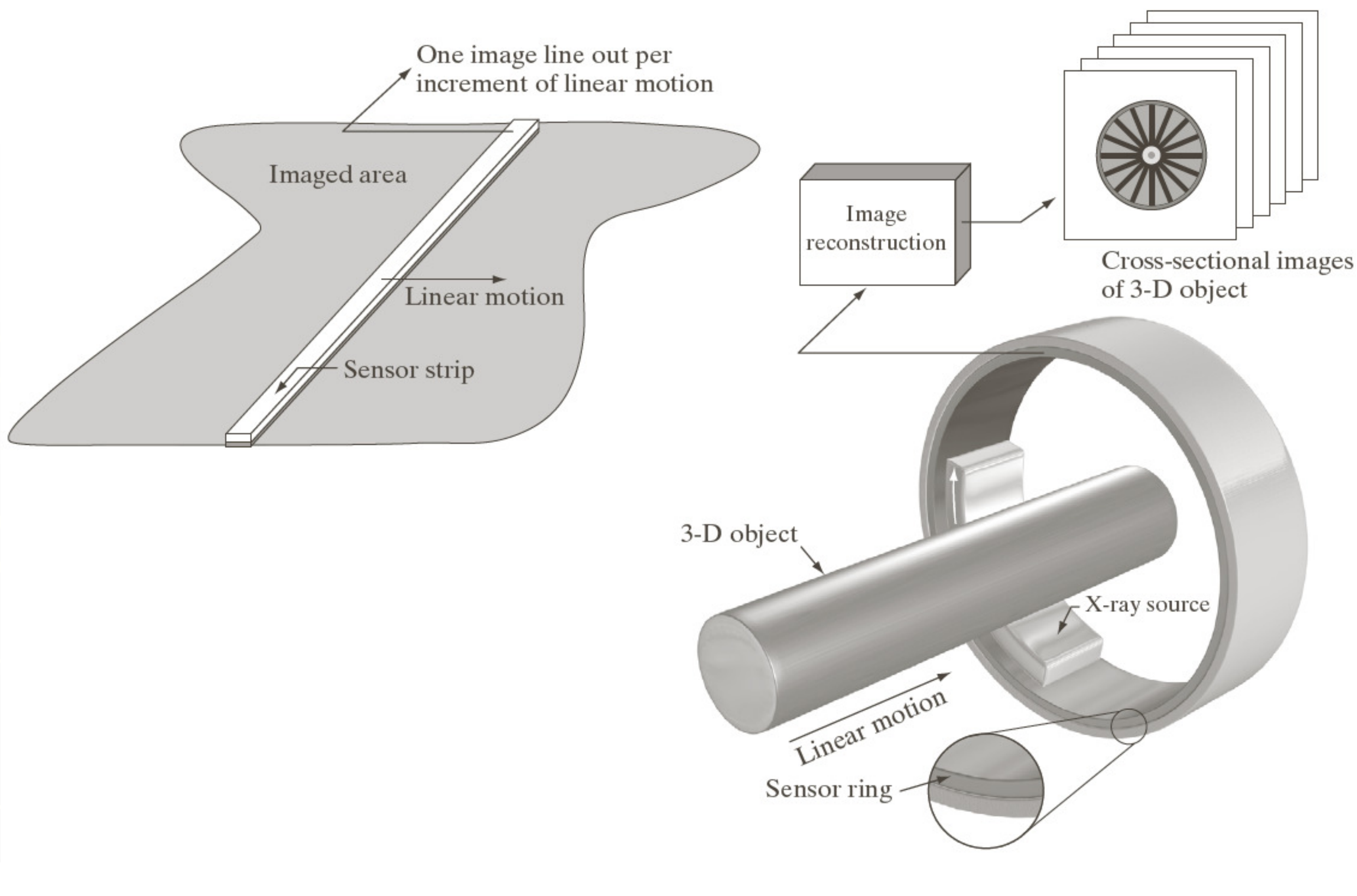



**FIGURE 2.14** (a) Image acquisition using a linear sensor strip. (b) Image acquisition using a circular sensor strip.

# Image Acquisition Process

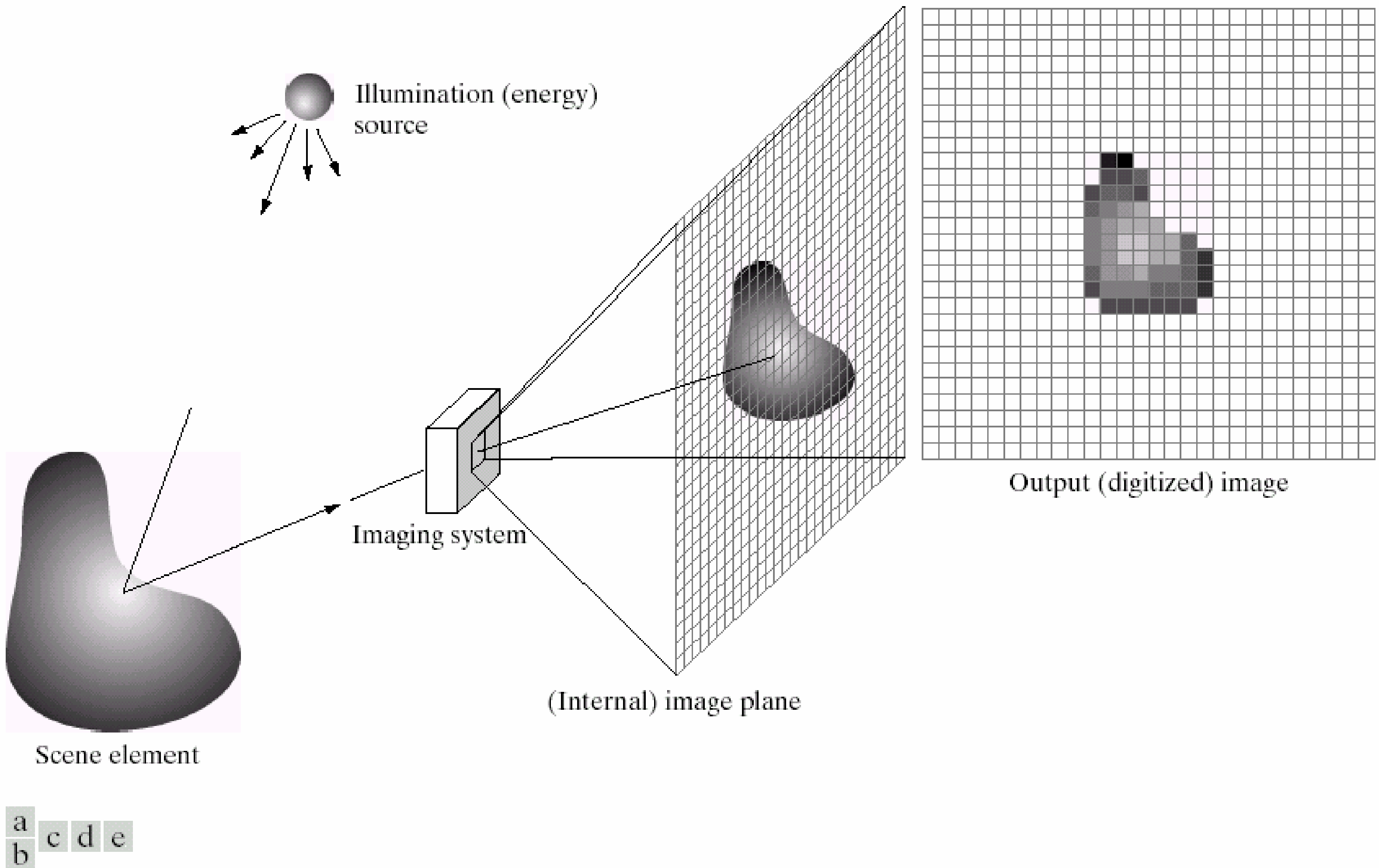



**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

# A Simple Image Formation Model

$$f(x, y) = i(x, y) \cdot r(x, y)$$

$f(x, y)$: intensity at the point $(x, y)$

$i(x, y)$: illumination at the point $(x, y)$

(the amount of source illumination incident on the scene)

$r(x, y)$: reflectance/transmissivity at the point $(x, y)$

(the amount of illumination reflected/transmitted by the object)

where $0 < i(x, y) < \infty$ and $0 < r(x, y) < 1$

# Some Typical Ranges of illumination

- Illumination

  Lumen — A unit of light flow or luminous flux

  Lumen per square meter ($lm/m^2$) — The metric unit of measure for illuminance of a surface

  - On a clear day, the sun may produce in excess of 90,000 $lm/m^2$ of illumination on the surface of the Earth
  - On a cloudy day, the sun may produce less than 10,000 $lm/m^2$ of illumination on the surface of the Earth
  - On a clear evening, the moon yields about 0.1 $lm/m^2$ of illumination
  - The typical illumination level in a commercial office is about 1000 $lm/m^2$

# Some Typical Ranges of Reflectance

- Reflectance
  - 0.01 for black velvet
  - 0.65 for stainless steel
  - 0.80 for flat-white wall paint
  - 0.90 for silver-plated metal
  - 0.93 for snow

# Image Sampling and Quantization

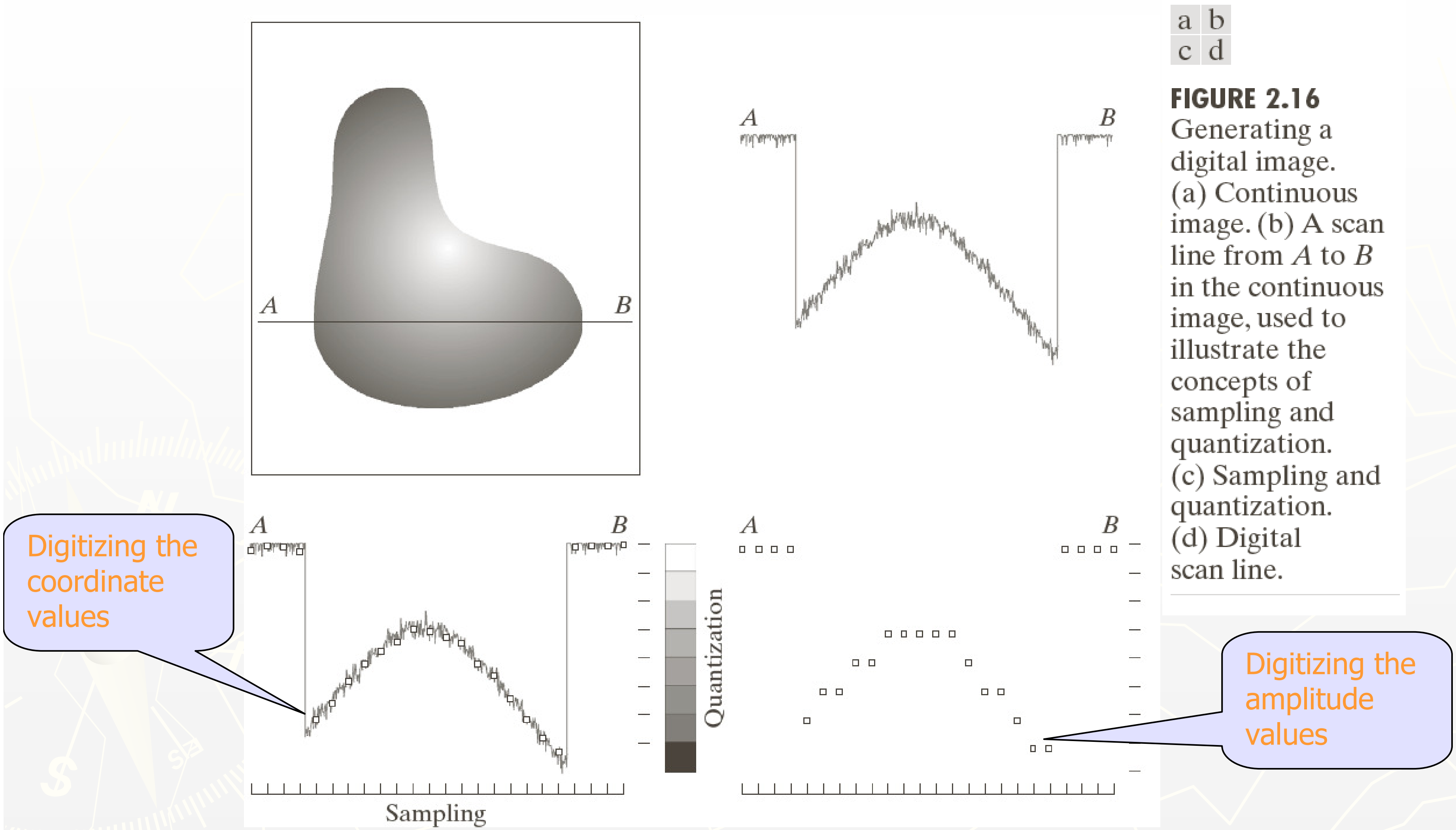
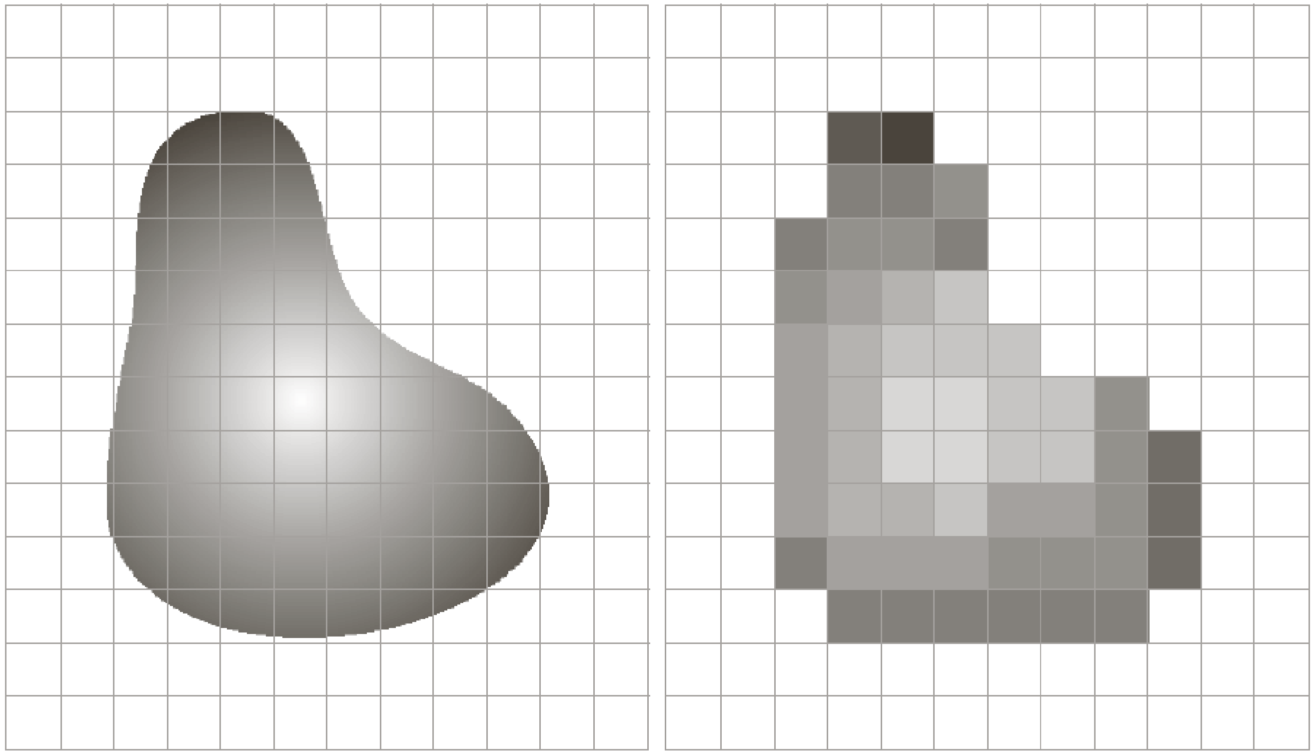



**FIGURE 2.16** Generating a digital image. (a) Continuous image. (b) A scan line from *A* to *B* in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

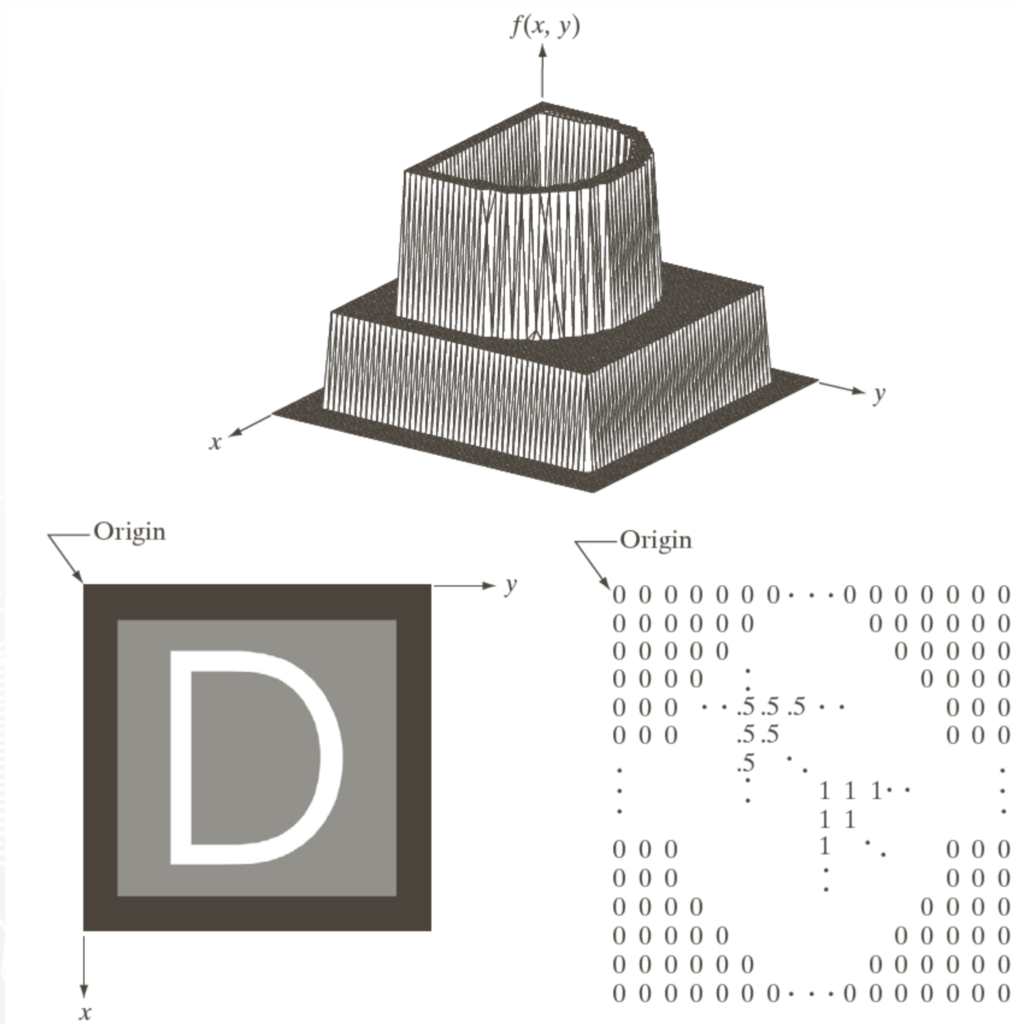
# Image Sampling and Quantization



a b

**FIGURE 2.17** (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.

# Representing Digital Images




a
b c

**FIGURE 2.18**
(a) Image plotted as a surface.
(b) Image displayed as a visual intensity array.
(c) Image shown as a 2-D numerical array (0, .5, and 1 represent black, gray, and white, respectively).

# Representing Digital Images

- The representation of an M×N numerical array as

$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \dots & \dots & \dots & \dots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

# Representing Digital Images

- The representation of an M×N numerical array as

$$A = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{M-1,0} & a_{M-1,1} & \cdots & a_{M-1,N-1} \end{bmatrix}$$

# Representing Digital Images

- The representation of an M×N numerical array in MATLAB

$$f(x, y) = \begin{bmatrix} f(1,1) & f(1,2) & \dots & f(1,N) \\ f(2,1) & f(2,2) & \dots & f(2,N) \\ \dots & \dots & \dots & \dots \\ f(M,1) & f(M,2) & \dots & f(M,N) \end{bmatrix}$$

# Representing Digital Images

- Discrete intensity interval [0, L-1], $L=2^k$

- The number b of bits required to store a $M \times N$ digitized image

$$b = M \times N \times k$$

# Representing Digital Images

**TABLE 2.1**
Number of storage bits for various values of $N$ and $k$.

| $N/k$ | 1 ($L$ = 2) | 2 ($L$ = 4) | 3 ($L$ = 8) | 4 ($L$ = 16) | 5 ($L$ = 32) | 6 ($L$ = 64) | 7 ($L$ = 128) | 8 ($L$ = 256) |
|---|---|---|---|---|---|---|---|---|
| 32 | 1,024 | 2,048 | 3,072 | 4,096 | 5,120 | 6,144 | 7,168 | 8,192 |
| 64 | 4,096 | 8,192 | 12,288 | 16,384 | 20,480 | 24,576 | 28,672 | 32,768 |
| 128 | 16,384 | 32,768 | 49,152 | 65,536 | 81,920 | 98,304 | 114,688 | 131,072 |
| 256 | 65,536 | 131,072 | 196,608 | 262,144 | 327,680 | 393,216 | 458,752 | 524,288 |
| 512 | 262,144 | 524,288 | 786,432 | 1,048,576 | 1,310,720 | 1,572,864 | 1,835,008 | 2,097,152 |
| 1024 | 1,048,576 | 2,097,152 | 3,145,728 | 4,194,304 | 5,242,880 | 6,291,456 | 7,340,032 | 8,388,608 |
| 2048 | 4,194,304 | 8,388,608 | 12,582,912 | 16,777,216 | 20,971,520 | 25,165,824 | 29,369,128 | 33,554,432 |
| 4096 | 16,777,216 | 33,554,432 | 50,331,648 | 67,108,864 | 83,886,080 | 100,663,296 | 117,440,512 | 134,217,728 |
| 8192 | 67,108,864 | 134,217,728 | 201,326,592 | 268,435,456 | 335,544,320 | 402,653,184 | 469,762,048 | 536,870,912 |

# Spatial and Intensity Resolution

- Spatial resolution
  - — A measure of the smallest discernible detail in an image
  - — stated with *line pairs per unit distance, dots (pixels) per unit distance, dots per inch (dpi)*

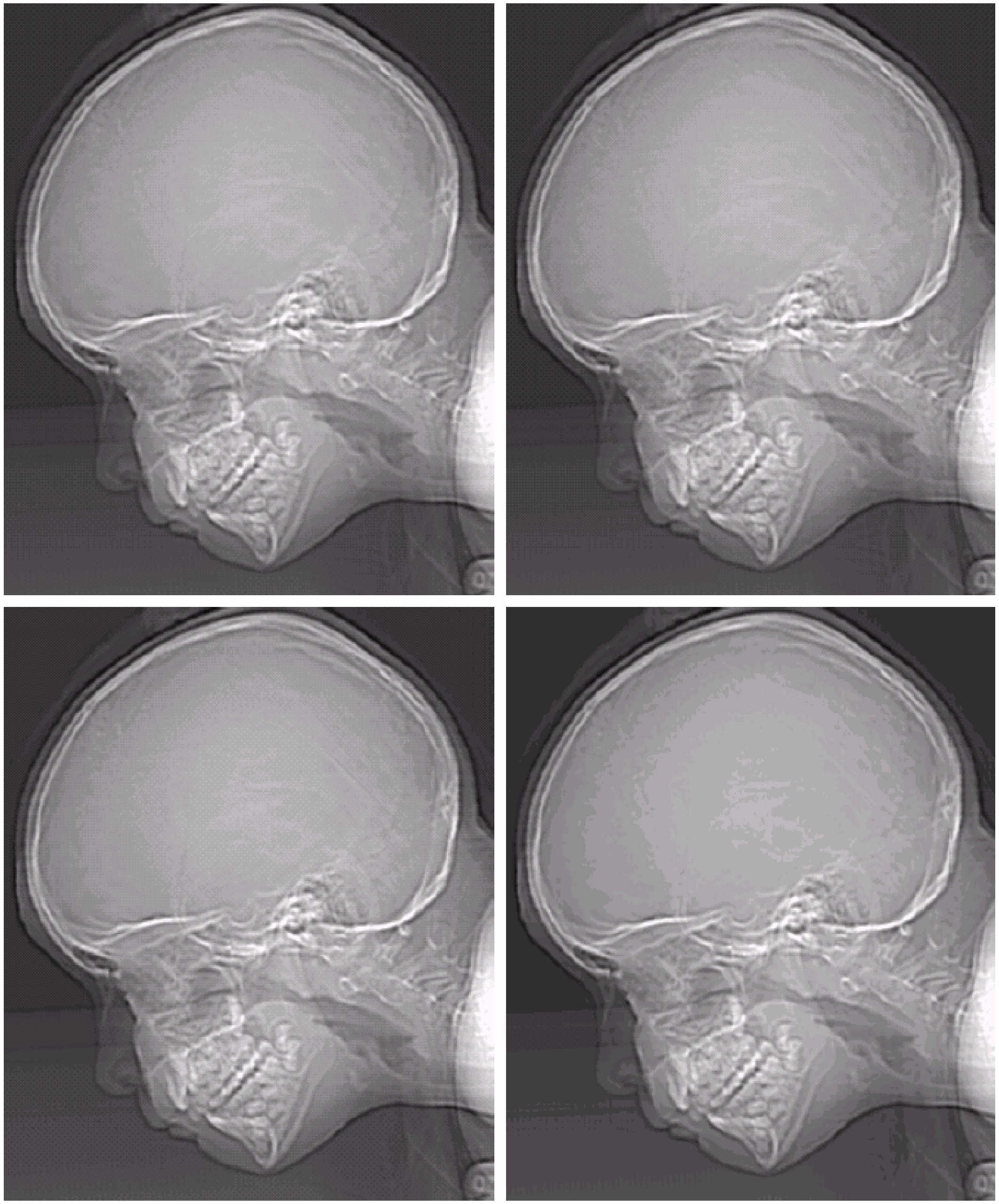
- Intensity resolution
  - — The smallest discernible change in intensity level
  - — stated with *8 bits, 12 bits, 16 bits, etc.*

# Spatial and Intensity Resolution

a b
c d

**FIGURE 2.20** Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.

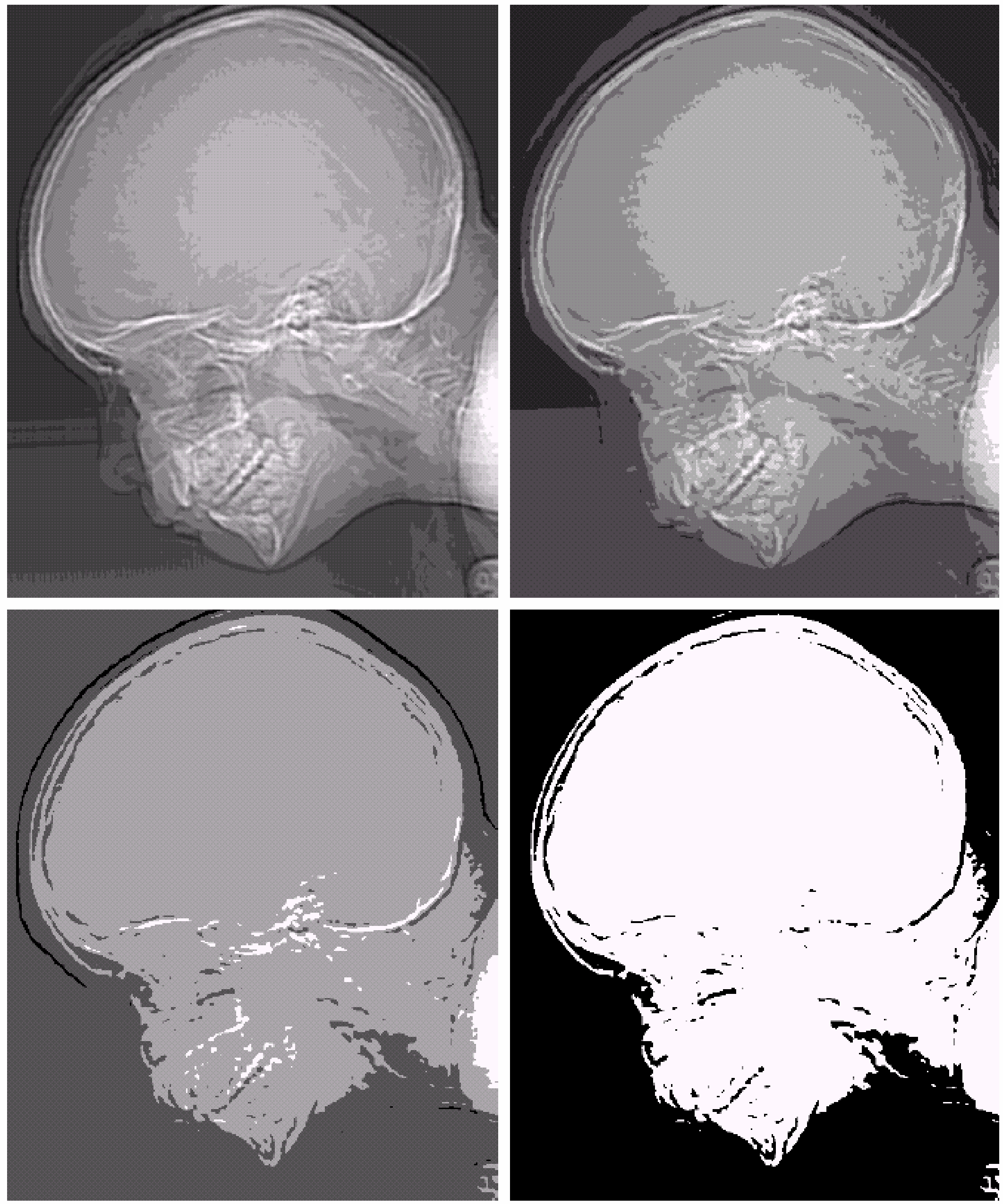
# Spatial and Intensity Resolution



a b
c d

**FIGURE 2.21**
(a) 452 × 374, 256-level image. (b)–(d) Image displayed in 128, 64, and 32 gray levels, while keeping the spatial resolution constant.

# Spatial and Intensity Resolution



e f
g h

**FIGURE 2.21** *(Continued)* (e)–(h) Image displayed in 16, 8, 4, and 2 gray levels. (Original courtesy of Dr. David R. Pickens, Department of Radiology & Radiological Sciences, Vanderbilt University Medical Center.)

# Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

  *e.g.,* zooming, shrinking, rotating, and geometric correction

- **Interpolation** (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

  Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

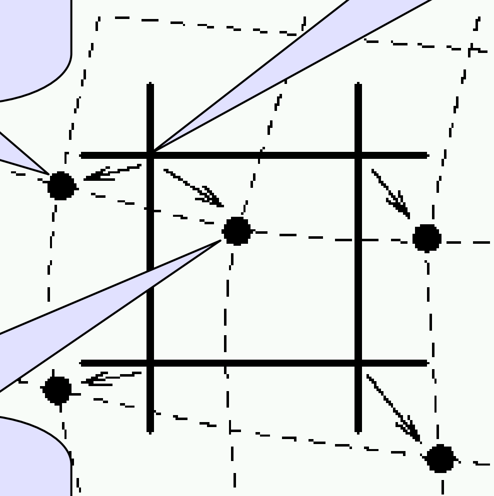
  http://www.dpreview.com/learn/?/key=interpolation

# Image Interpolation:

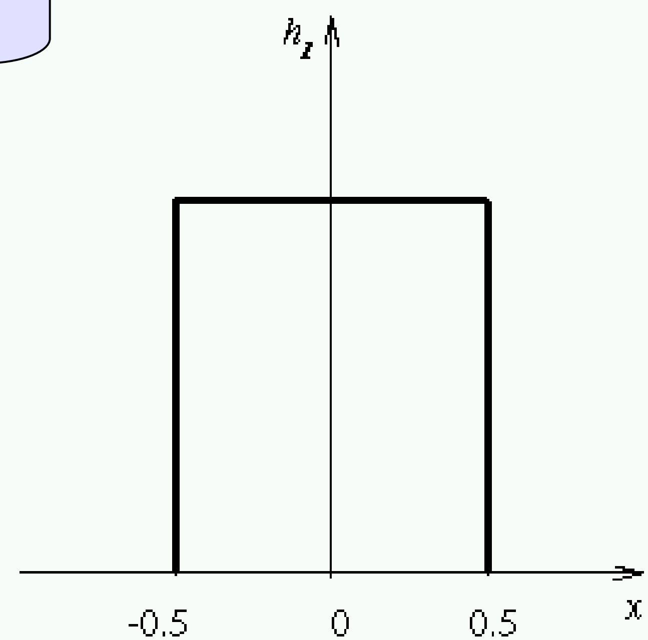
# Nearest Neighbor Interpolation

$f_1(x_2,y_2) =$
$f(\text{round}(x_2), \text{round}(y_2))$
$=f(x_1,y_1)$

$f(x_1,y_1)$

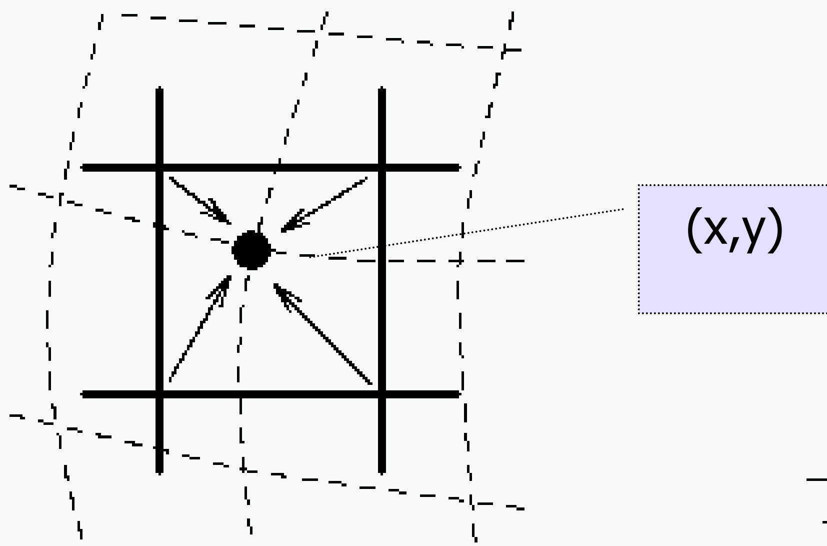


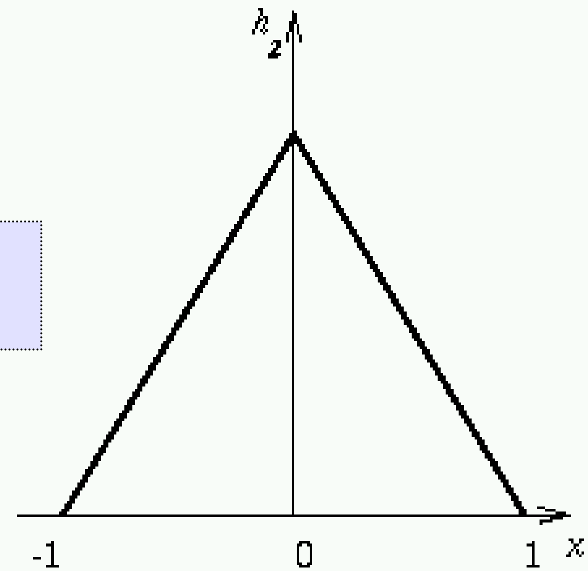
$f_1(x_3,y_3) =$
$f(\text{round}(x_3), \text{round}(y_3))$
$=f(x_1,y_1)$



# Image Interpolation:

# Bilinear Interpolation




$$
\begin{aligned}
& f_2(x, y) \\
&= (1-a)\cdot(1-b)\cdot f(l,k) + a\cdot(1-b)\cdot f(l+1,k) \\
&+(1-a)\cdot b\cdot f(l,k+1) + a\cdot b\cdot f(l+1,k+1) \\
& l = floor(x), k = floor(y), a = x-l, b = y-k.
\end{aligned}
$$

# Image Interpolation: Bicubic Interpolation

- The intensity value assigned to point (x,y) is obtained by the following equation

$$f_3(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x^i y^j$$

- The sixteen coefficients are determined by using the sixteen nearest neighbors.

http://en.wikipedia.org/wiki/Bicubic_interpolation

# Examples: Interpolation

# Examples: Interpolation

Nearest Neighbor Interpolation

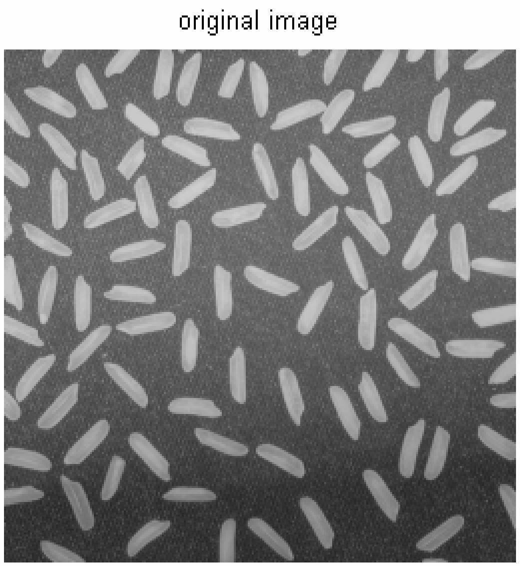
# Examples: Interpolation

Bilinear Interpolation

# Examples: Interpolation

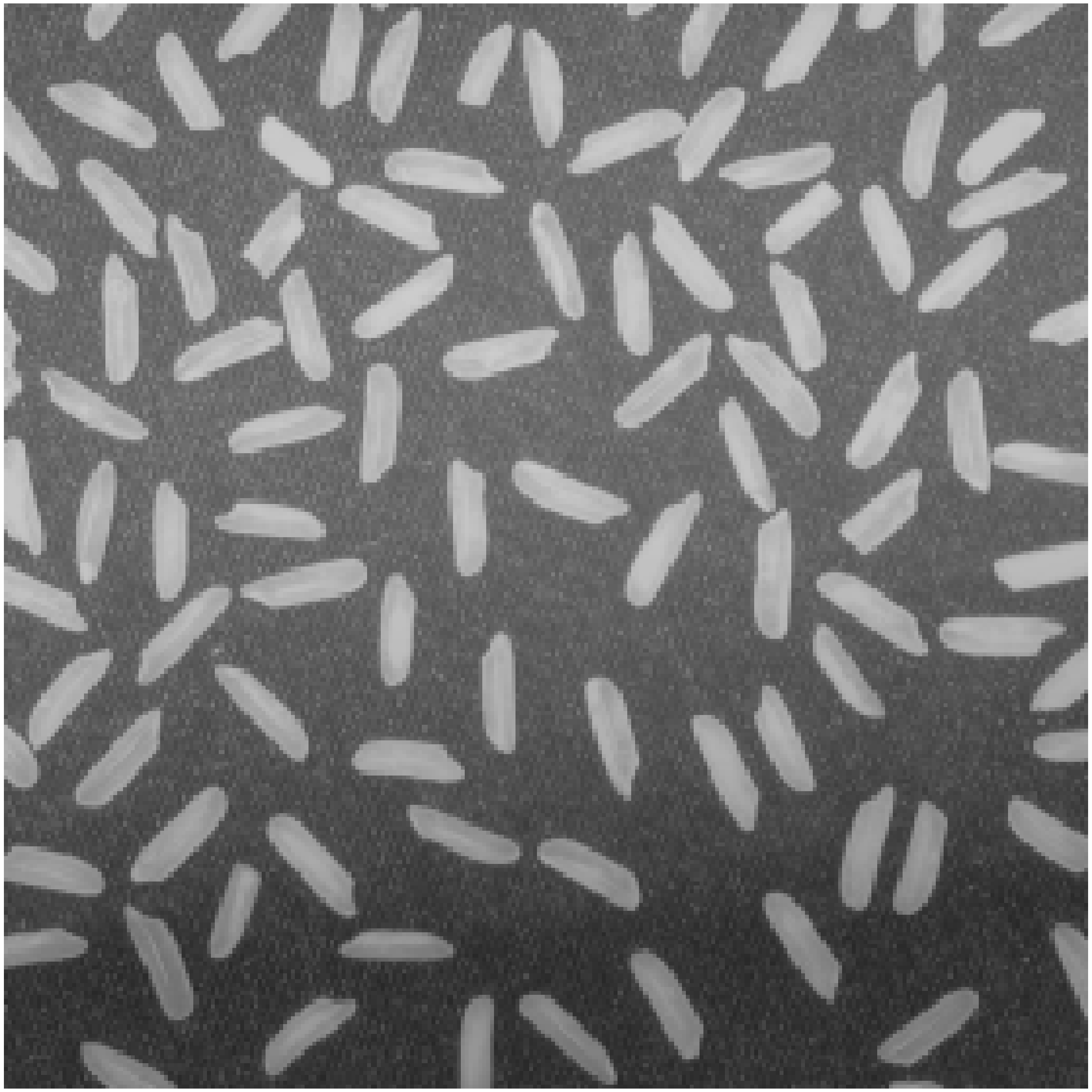
Bicubic Interpolation

# Examples: Interpolation



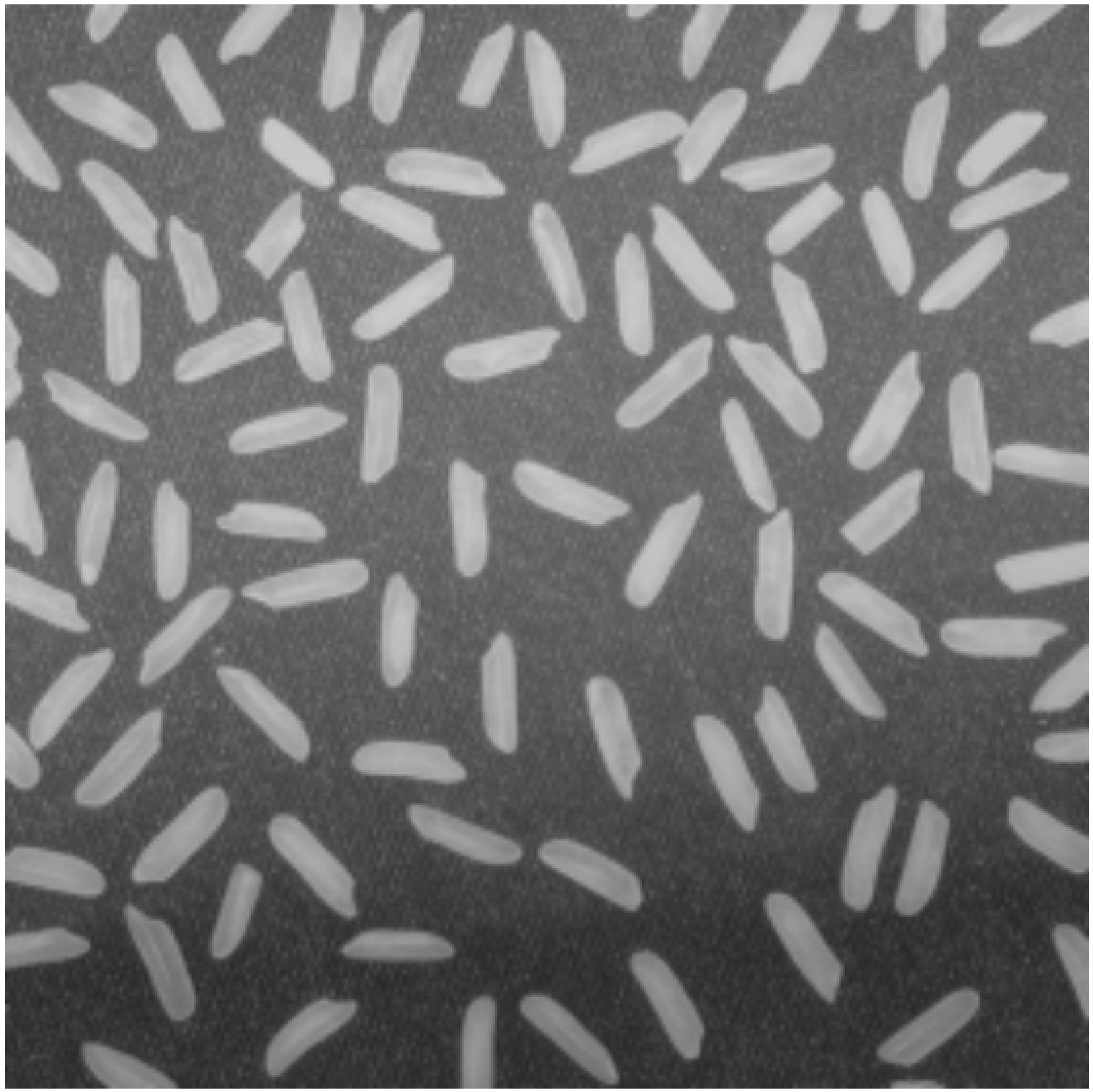
# Examples: Interpolation

nearest



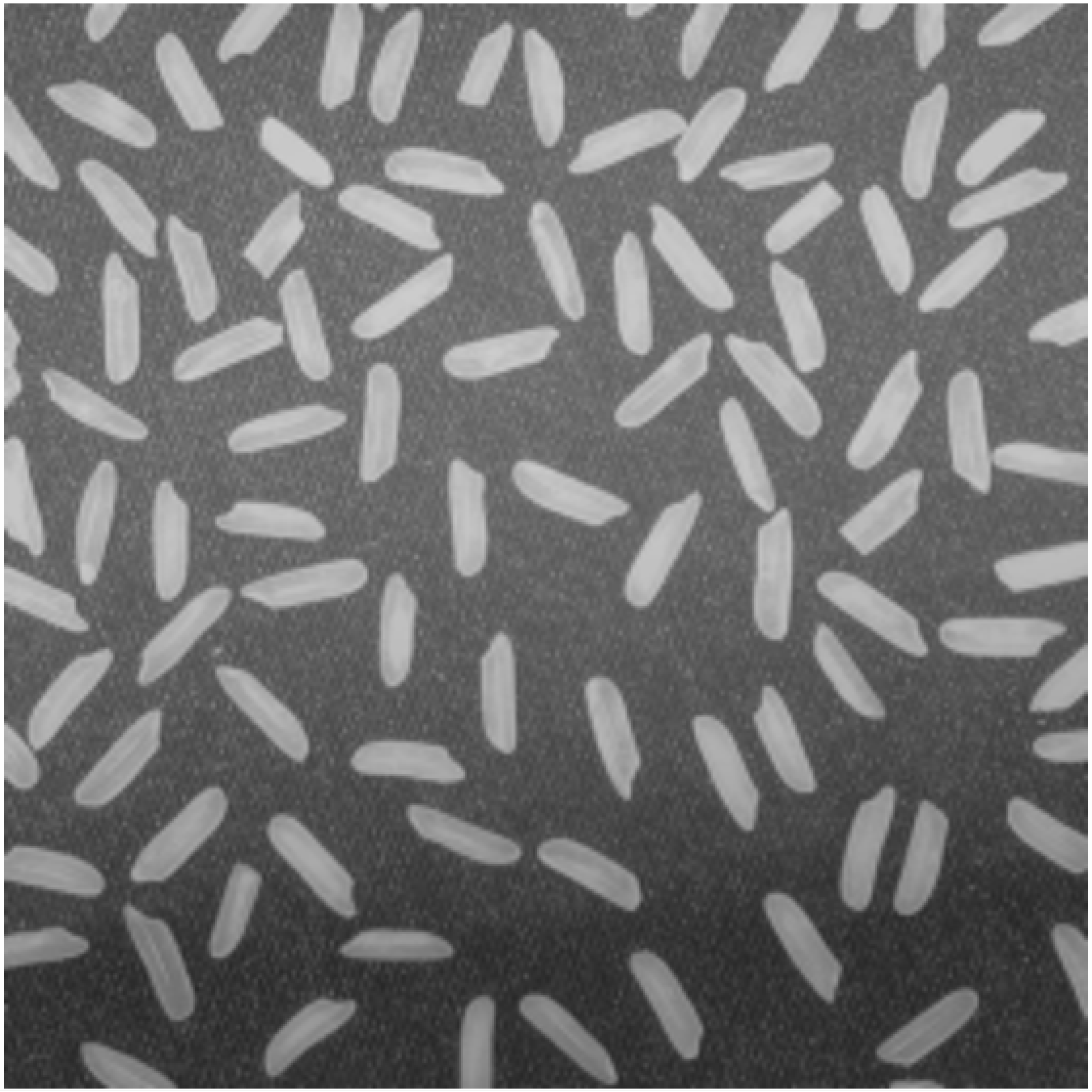
# Examples: Interpolation

bilinear



# Examples: Interpolation

bicubic



# Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

# Basic Relationships Between Pixels

- **Neighbors** of a pixel p at coordinates (x,y)

- **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
  (x-1, y), (x+1, y), (x,y-1), and (x, y+1).

- **4 diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
  (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).

- **8 neighbors of p**, denoted $\mathbf{N_8(p)}$
  $N_8(p) = N_4(p) \cup N_D(p)$

# Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

# Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **m-adjacency**: Two pixels p and q with values from V are m-adjacent if

  (i) q is in the set $N_4(p)$, or

  (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

# Basic Relationships Between Pixels

- **Path**
- A (digital) path (or curve) from pixel p with coordinates $(x_0, y_0)$ to pixel q with coordinates $(x_n, y_n)$ is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where $(x_i, y_i)$ and $(x_{i-1}, y_{i-1})$ are adjacent for $1 \le i \le n$.

- Here $n$ is the *length* of the path.
- If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.
- We can define 4-, 8-, and m-paths based on the type of adjacency used.

# Examples: Adjacency and Path

**V = {1, 2}**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | | 0 | 1 | 1 | | 0 | 1 | 1 |
| 0 | 2 | 0 | | 0 | 2 | 0 | | 0 | 2 | 0 |
| 0 | 0 | 1 | | 0 | 0 | 1 | | 0 | 0 | 1 |

# Examples: Adjacency and Path

**V = {1, 2}**

| | | |
|---|---|---|
| 0 | 1 | 1 |
| 0 | 2 | 0 |
| 0 | 0 | 1 |

| | | |
|---|---|---|
| 0 | 1 | 1 |
| 0 | 2 | 0 |
| 0 | 0 | 1 |

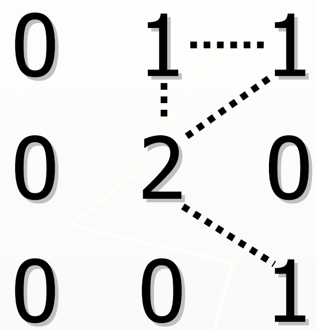
| | | |
|---|---|---|
| 0 | 1 | 1 |
| 0 | 2 | 0 |
| 0 | 0 | 1 |

**8-adjacent**

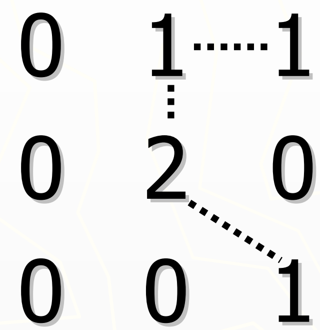
# Examples: Adjacency and Path

**V = {1, 2}**

| | | |
|---|---|---|
| 0 | 1 | 1 |
| 0 | 2 | 0 |
| 0 | 0 | 1 |




**8-adjacent**

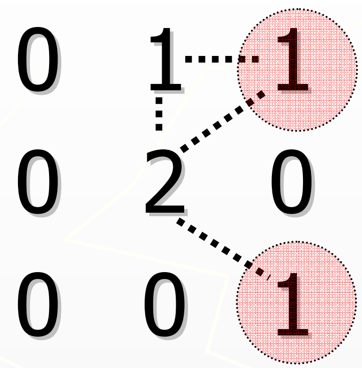



**m-adjacent**

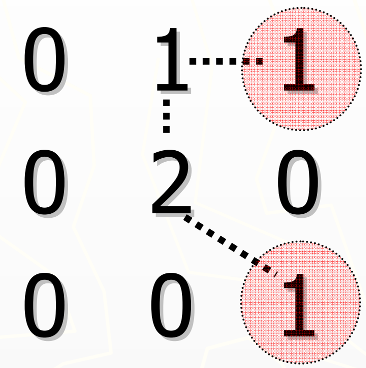
# Examples: Adjacency and Path

**V = {1, 2}**

| | | |
|---|---|---|
| $0_{1,1}$ | $1_{1,2}$ | $1_{1,3}$ |
| $0_{2,1}$ | $2_{2,2}$ | $0_{2,3}$ |
| $0_{3,1}$ | $0_{3,2}$ | $1_{3,3}$ |




**8-adjacent**




**m-adjacent**

The 8-path from (1,3) to (3,3):
(i) (1,3), (1,2), (2,2), (3,3)
(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):
(1,3), (1,2), (2,2), (3,3)

# Basic Relationships Between Pixels

- **Connected in S**

  Let S represent a subset of pixels in an image. Two pixels p with coordinates $(x_0, y_0)$ and q with coordinates $(x_n, y_n)$ are said to be **connected in S** if there exists a path

  $$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

  Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

# Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.
- If S has only one connected component, then S is called ***Connected Set***.
- We call R a **region** of the image if R is a connected set
- Two regions, $R_i$ and $R_j$ are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not to be adjacent are said to be ***disjoint.***

# Basic Relationships Between Pixels

- **Boundary (or border)**

  - The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
  - If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

- **Foreground and background**

  - An image contains K disjoint regions, $R_k$, $k = 1, 2, \ldots, K$. Let $R_u$ denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

  All the points in $R_u$ is called **foreground**;

  All the points in $(R_u)^c$ is called **background**.